Chapter 6 / **Example 10**

Modelling with polynomials

The path of a ski jumper is illustrated on the diagram on the right.
In order to determine a function to model this jump, the diagram was processed with graphing software to determine a set of points that the path of the ski jumper goes through, x being the horizontal distance from the starting point and y being the vertical displacement from the starting point. The points are listed in the table on the next page.

x	34.8	58.2	83.1	105.9	120.3	141.9	161.4	180	195	206.1	218.4	228.9	239.4
y	–19.5	–34.2	–49.5	–61.8	–64.8	–64.8	–68.1	–74.4	–82.5	–90.6	–101.1	–112.5	–125.7

a Enter the data into your GDC.
b What type of function would model this set of data points and why?
c Use your GDC to determine the model function for this set of data.
d Assess the choice of model by determining the coefficient of determination.
e Plot the model function over the scatter plot and comment on the closeness of fit to the original data.

The straight line that represents the slope of the mountain on which the skier will land goes through the points (0, 0) and (360, -210).

f Determine the equation of the line going through these two points.
g Find the point at which the ski jumper will land on the slope.

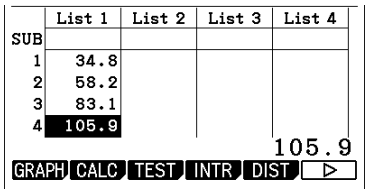
From that point on and up to the point where $x = 400$, the skier moves along the straight line describing the slope of the mountain.

h Write in piecewise form the function describing the whole path of the skier.

Press MENU 2 STAT to display the List Editor screen.

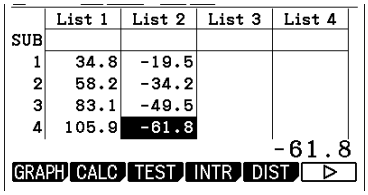
Enter the x-coordinates in the first column.

Press EXE after each number to move to the next cell.



Press ▶ to move to the next column.

Enter the y-coordinates in the second column.



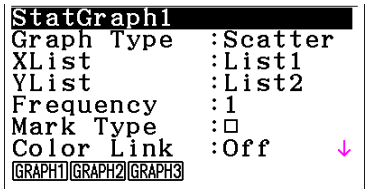
Chapter 6 / **Example 10**

Modelling with polynomials

Press [F1] GRAPH.

Press [F6] SET.

Choose Graph Type: [F1] Scatter, XList: List1 and YList: List2.

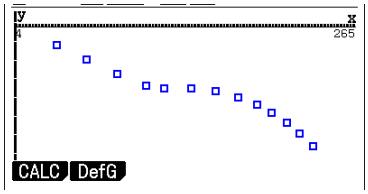


Press [EXIT].

Press [F1] GRAPH1.

The GDC displays a scatter diagram of x against y.

Because the data is approximately cubic, cubic regression is appropriate.

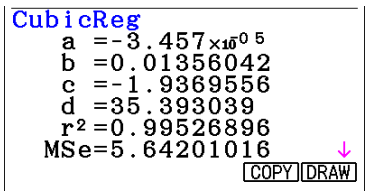


To calculate the equation of the regression line

Press [F1] CALC, press [F5] X^3.

The cubic curve is given by the equation $y = -0.0000346x^3 + 0.0136x^2 - 1.94x + 35.4$.

The coefficient of determination is $R^2 = 0.995$, which shows very strong cubic association.

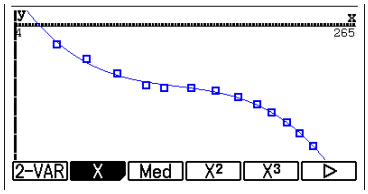


Press [F5] COPY.

The GDC displays the equation entry screen.

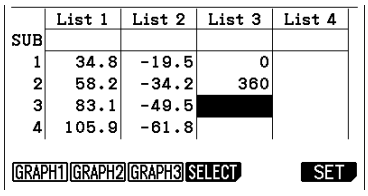
Press [SHIFT] [9] PASTE and press [EXE].

Press [F6] DRAW.

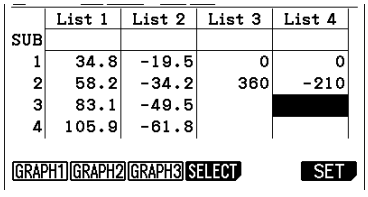


Press [EXIT] twice to return to the list editor.

Move to the next available column and enter the x-coordinates: 0 and 360 in L3.



Move to the next available column and enter the y-coordinates: 0 and –210 in L4.



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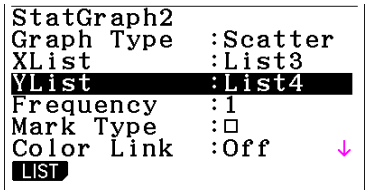
Press [F6] SET.

Press [F2] GRAPH2.

Choose Graph Type: [F1] Scatter, XList: List3 and YList: List4

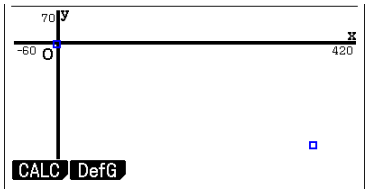
To enter List 3 and 4, press [F1] List and then type 3 and 4.



Press [EXIT].

Press [F1] GRAPH1.

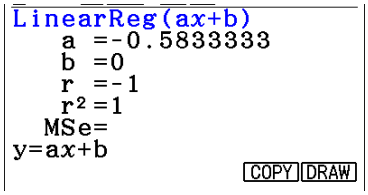
The GDC displays the two points.



To find the equation of the line through these two points, calculate the equation of the regression line. Press [F1] CALC, [F2] X, [F1] ax+b.

The form of the regression equation is '$y = ax + b$'.

The straight line is given by the equation $y = -0.583x$.

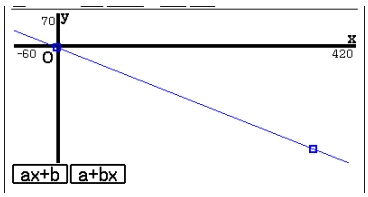


Press [F5] COPY.

The GDC displays the equation entry screen.

Press [SHIFT] [9] PASTE and press [EXE] to paste the function in Y2.

Press [F6] DRAW.



Choose suitable window settings to display the piecewise graph.

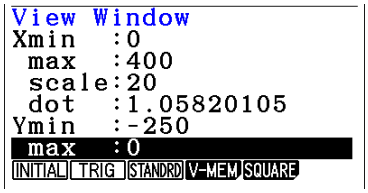
Press [MENU] 5 [GRAPH] ⏸

Press [F1] SELECT to select both Y1 and Y2.

Press [SHIFT] [F3] V-WIN.

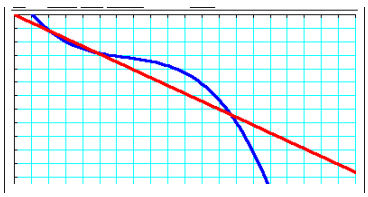
Set the axes to show $0 \le x \le 400$ and $-250 \le y \le 0$ with scales of 20. Leave the remaining items the same.

Press [EXIT] when you have finished.



Press [F6] DRAW to display the graph screen.

The GDC displays the two functions in a suitable window.



Chapter 6 / **Example 10**

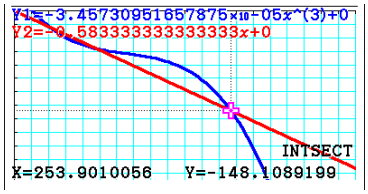
Modelling with polynomials

To find the intersection press [F5] G-Solv [F5] Intersect.

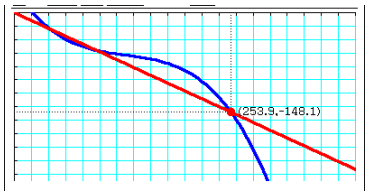
Press ▶ twice to move to the required intersection.

Press [EXE] to display the coordinates.

Press [EXIT] to leave G-Solv mode and [F6] DRAW to display the graph screen again.



The GDC displays the intersection of the curve and the straight line at the point $(254, -148)$.



The piecewise function is

$$f(x) = \begin{cases} -0.0000346x^3 + 0.0136x^2 - 1.94x + 35.4, & 0 \le x \le 254 \\ -0.583x, & 254 < x \le 400 \end{cases}$$

Press [EXIT] to display the equation entry screen.

Modify Y1 by adding ,[0,254] and Y2 by adding ,[254,400]

To edit a function press [SHIFT] [8] CLIP, [F1] CPY-LINE and [SHIFT] [9] PASTE, edit the function and press [EXE].

Press [F6] DRAW.